Factorization

📌 **Definition:**

Factorization is the process of rewriting an expression as a product of simpler expressions.

📌 **Common Methods:**
Factoring out the Greatest Common Divisor (GCD)

$$ax + ay = a(x + y)$$

📌 **Key Fact:**
✅ **Factorization is the reverse of expansion.**

Example Problem
Factorize:

$$6x + 9$$

$$6x+9 = 3(2x)+3(3)$$

We reveal the common divisor (3)

$$= 3(2x+3)$$

Difference of Squares Formula

$$(a - b)(a + b) = a^2 - b^2$$

It's always the negative sign that appears here

The product of the sum and difference of two terms equals the difference of their squares.

Difference of Squares Formula

Simplify $(5 - 3)(5 + 3)$

📌 **Solution:**

Using the formula:

$$a^2 - b^2 = 5^2 - 3^2 = 25 - 9 = 16$$

📢 Final Answer: 16

Square of a Binomial

📌 **Definition:**

The square of a binomial expands into a trinomial.

One with three terms

Expanding a Squared Binomial:

📌 **Formula:**

$$(a-b)^2 = a^2 - 2ab + b^2$$

The sign here has to be the same one in the expanded form

Expand: $(x-4)^2$

a, b

Remember!

- Square of the first term: a^2
- Twice the product of the first and second terms: 2ab
- Square of the second term: b^2

📌 **Solution:**

Using the formula:

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(x-4)^2 = x^2 - 2(x)(4) + 4^2$$

Term 1, Term 2, Term 3 → three terms = trinomial

$$= x^2 - 8x + 16$$

Expansion (Development)

📌 **Definition:**

Expansion is the process of removing parentheses by multiplying terms.

📌 **Common Expansion Rules:**

1- Distributive Property:

$$a(b+c) = ab + ac$$

2- Binomial Square:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

3- Product of Binomials:

$$(a+b)(c+d) = ac + ad + bc + bd$$

Example Problem

Expand:

$$(2x-3)(x+4)$$

📌 **Solution:**

Using distributive property:

$$2x \cdot x + 2x \cdot 4 - 3 \cdot x - 3 \cdot 4$$

$$= 2x^2 + 8x - 3x - 12$$

$$= 2x^2 + 5x - 12$$

We combine terms that have the same variable and the same exponent (degree)

For example, we combine X's together, and X^2's together

📢 Final Answer: $2x^2 + 5x - 12$

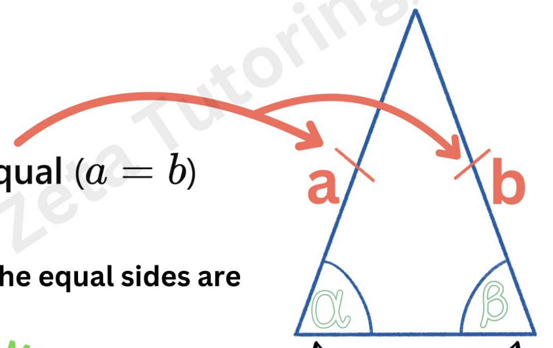
Isosceles Triangle

📌 **Definition:**

An isosceles triangle has at least **two equal sides** and **two equal angles**.

📌 **Key Properties:**

- Two sides are **equal** $(a = b)$



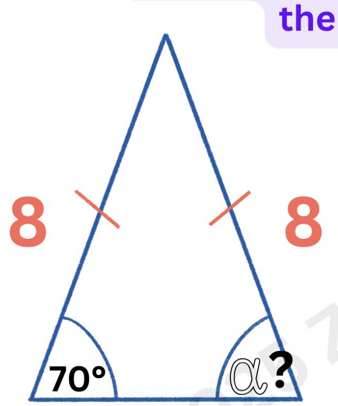
- **Two angles opposite the equal sides are** **Congruent:**

Meaning are equal

$(\alpha = \beta)$

ALPHA AND BETA ARE EQUAL ANGLES !

If this triangle has two sides equal to 8, calculate the measure of the Alpha angle:



Answer

Since this triangle has **two equal sides**, the angles opposite these sides **must also be equal**

Thus, the alpha angle is equal to the 70° angle. **α = 70°**

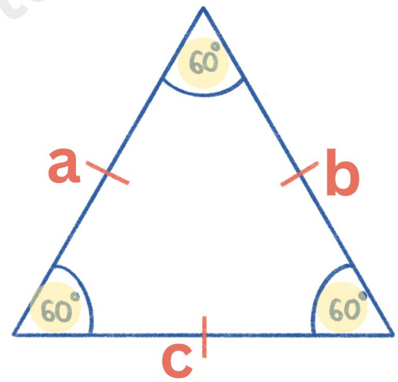
Equilateral Triangle

📌 **Definition:**

An equilateral triangle has **all three sides equal** and **all angles equal to 60°**.

📌 **Key Properties:**

- $a = b = c$
- All angles are equal to 60°



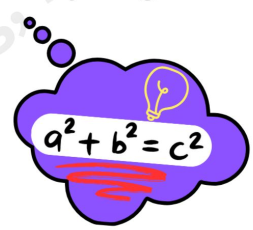
Right Triangle

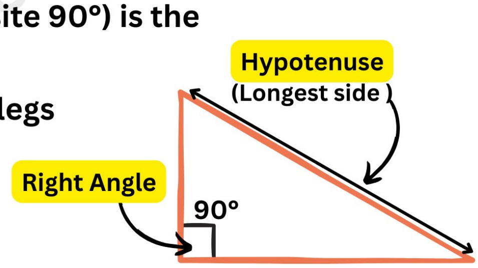
📌 **Definition:**

A right triangle has **one 90° angle** and follows the Pythagorean Theorem.

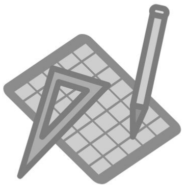
📌 **Key Properties:**

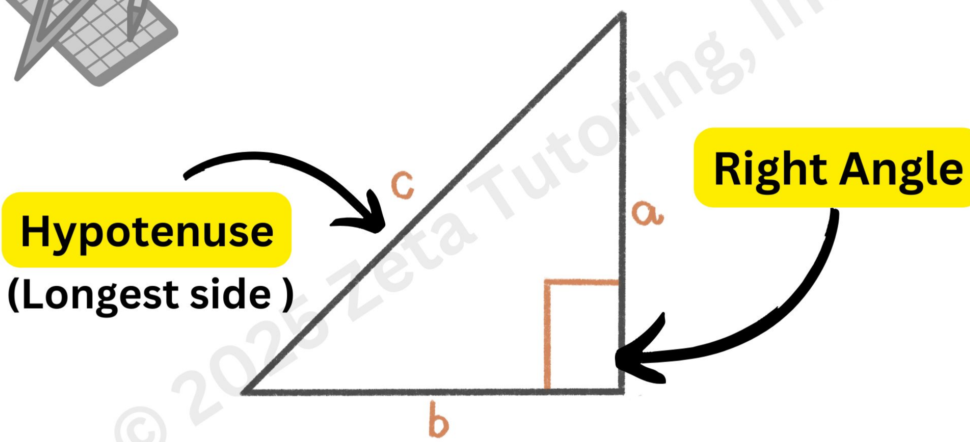
- **One angle is 90°**
- **The longest side (opposite 90°) is the hypotenuse**
- **The other two sides are legs**





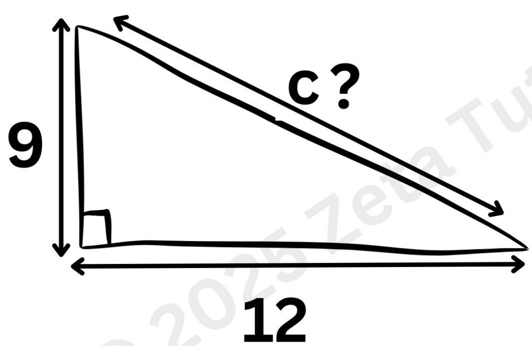
Pythagoras Theorem





Pythagoras Theorem

$$a^2 + b^2 = c^2$$



$$C^2 = 9^2 + 12^2$$
$$C^2 = 81 + 144$$
$$C^2 = 225$$
$$C = \sqrt{225}$$
$$C = 15$$

Area of a Triangle

📌 **Formula:**

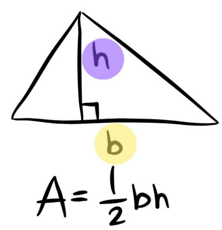
$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

✅ The area of a triangle is found by multiplying the base by the height and dividing by 2.

📌 **Example:**

For a triangle with base 6 cm and height 4 cm:

$$A = \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$$



Perimeter of a Triangle

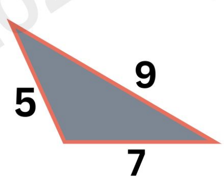
📌 **Formula:**

$$P = a + b + c$$

✅ The perimeter of a triangle is the sum of all its sides.

📌 **Example:**

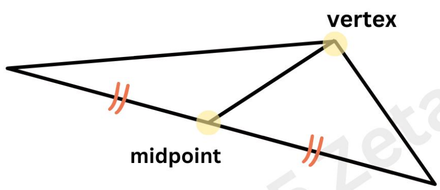
For a triangle with sides 5 cm, 7 cm, and 9 cm:

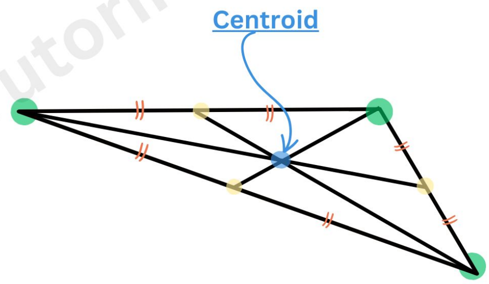


$$P = 5 + 7 + 9 = 21 \text{ cm}$$

Median (of a Triangle)

A line segment connecting a **vertex** of a triangle to the **midpoint** of the opposite side.

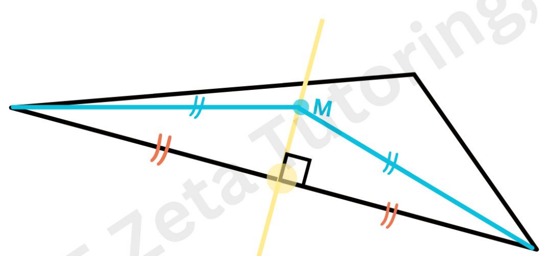




Property: The three medians of a triangle intersect at the centroid, which is the center of gravity of the triangle.

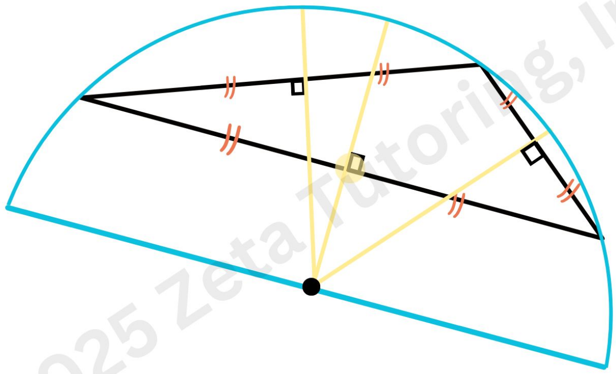
Perpendicular Bisector (Mediatrix)

A line that cuts a segment into **two equal parts** at a **90° angle**.



Property: Any point on the perpendicular bisector is **equidistant** from the two **endpoints of the segment**.

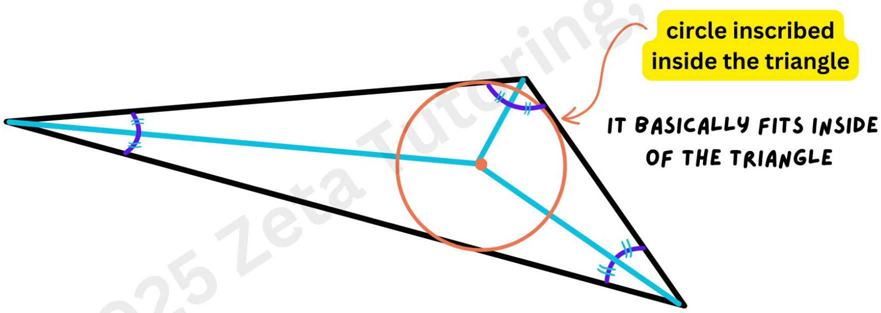
Perpendicular Bisector (Mediatrix)



Intersection Point: The three perpendicular bisectors of a triangle meet at the circumcenter, which is the **center of the circle that passes through all three vertices of the triangle**.

Angle Bisector

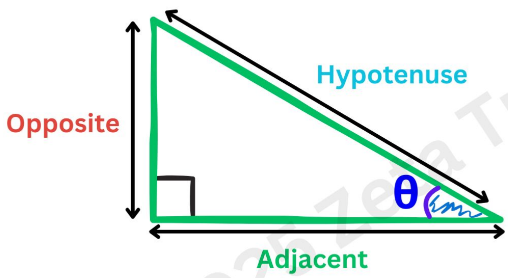
It's a line that divides an angle into two equal parts.



Property: Any point on the angle bisector is equidistant from the sides of the angle.

Sine, Cosine, and Tangent (Trigonometry Basics)

In a right-angled triangle, the three main trigonometric ratios are:



Sin θ = $\dfrac{\text{Opposite}}{\text{Hypotenuse}}$ **(SOH)**

Cos θ = $\dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ **(CAH)**

Tan θ = $\dfrac{\text{Opposite}}{\text{Adjacent}}$ **(TOA)**

Opposite: **The side opposite to the angle θ.**

Adjacent: **The side next to the angle θ (but not the hypotenuse).**

Hypotenuse: **The longest side of the right triangle (opposite the 90° angle).**

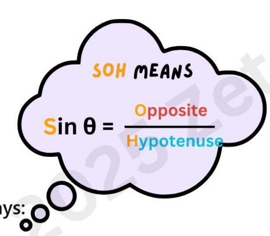
Sine, Cosine, and Tangent (Trigonometry Basics)

✅ Example 1: Finding a Side Using Sine

In a right triangle, if $\theta = 30^\circ$ and the hypotenuse is 10:

Find the opposite side:

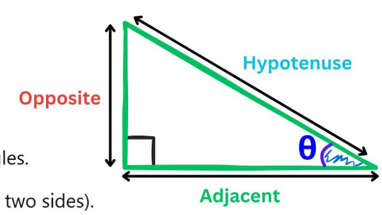
$$\sin 30^\circ = \frac{\text{opposite}}{10}$$



$$\text{opposite} = 10 \times \sin 30^\circ = 10 \times 0.5 = 5$$

💡 Key Takeaways:

- **Use SOH-CAH-TOA** to find sides and angles.
- **Inverse trigonometric functions** ($\sin^{-1}, \cos^{-1}, \tan^{-1}$) find angles.
- **Always check if you have enough info** (one angle & one side or two sides).



Exponents

📌An **Exponent** tells you **how many times to multiply a number by itself**.

Base → a^n ← Exponent or Power

Example: $2^3 = 2 \times 2 \times 2$

Product Rule of Exponents

📌 **Rule:**

$$a^m \times a^n = a^{m+n}$$

✅ **When multiplying same bases, add the exponents.**

📌 **Example:**

$$x^3 \times x^5 = x^{3+5} = x^8$$

🟩 **Example Problem:**

Simplify:

$$2^4 \times 2^3$$

📌 **Solution:** $2^{4+3} = 2^7$

$= 128$

Quotient Rule of Exponents

📌 **Rule:** $$\frac{a^m}{a^n} = a^{m-n}$$

✅ When dividing same bases, subtract the exponents.

📌 **Example:** $$\frac{x^7}{x^3} = x^{7-3} = x^4$$

🟩 **Example Problem:**

Simplify: $\frac{5^6}{5^2}$

📌 **Solution:** $5^{6-2} = 5^4$

$= 625$

Power of a Power Rule

📌 **Rule:** $$(a^m)^n = a^{m \times n}$$

✅ When raising a power to another power, multiply the exponents.

📌 **Example:** $$(x^3)^4 = x^{3\times 4} = x^{12}$$

🟩 **Example Problem:**

Simplify: $(2^5)^3$

📌 **Solution:** $2^{5\times 3} = 2^{15}$

$= 32,768$

Power of a Product Rule

📌 **Rule:** $$(ab)^m = a^m \times b^m$$

✅ When raising a product to a power, apply the exponent to each factor.

📌 **Example:**

$$(2x)^3 = 2^3 \times x^3 = 8x^3$$

🟩 **Example Problem:**

Simplify:

$$(3y)^4$$

📌 **Solution:**

$$3^4 \times y^4 = 81y^4$$

Negative Exponents Rule

📌 **Rule:**

$$a^{-m} = \frac{1}{a^m}$$

✅ A negative exponent means reciprocal.

The reciprocal of a number is simply "flipping" the fraction. In other words, the numerator and denominator switch places

📌 **Example:**

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

Zero Exponent Rule

📌 **Rule:**

$$a^0 = 1$$

✅ Any number raised to the power 0 is 1 (except 0^0, which is undefined).

📌 **Example:**

$$(10x)^0 = 1$$

Definition of a Radical

What is a Radical?

📌 **Definition:**

A radical (or root) is <u>the inverse operation of exponentiation</u>. The radical symbol (√) represents the principal square root, but it can also represent cube roots, fourth roots, etc.

📌 **General Form:**

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

- Square Root (n = 2): $\sqrt{a} = a^{\frac{1}{2}}$
- Cube Root (n = 3): $\sqrt[3]{a} = a^{\frac{1}{3}}$
- Higher Roots: $\sqrt[4]{a} = a^{\frac{1}{4}}, \quad \sqrt[5]{a} = a^{\frac{1}{5}}$, etc.

Definition of a Radical

What is a Radical?

📌 **Example:** $\sqrt[5]{32}$

Since $32 = 2^5$, we apply the radical rule:

$$\sqrt[5]{32} = \left(2^5\right)^{\frac{1}{5}} = 2^{\frac{5}{5}} = 2$$

A good question to ask is: What number, when multiplied by itself five times, results in 32?

YOU GUESSED IT, IT'S (2):
2 x 2 x 2 x 2 x 2 = 32!

Product Rule of Radicals

📌 **Rule:** $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

✅ **You can multiply radicals if they have the same index.**

📌 **Example:**

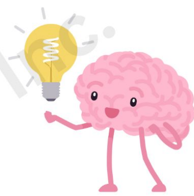
$$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$$

Both numbers are under the same radical sign (√)

Quotient Rule of Radicals

📌 **Rule:**

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, \quad b \neq 0$$



✅ You can divide radicals if they have the same index.

📌 **Example:**

$$\frac{\sqrt{25}}{\sqrt{5}} = \sqrt{\frac{25}{5}} = \sqrt{5}$$

Power Rule of Radicals

📌 **Rule:**

$$(\sqrt{a})^n = a^{\frac{n}{2}}$$

✅ A radical can be rewritten as a fractional exponent.

📌 **Example:**

$$\sqrt{9}^4 = 9^{\frac{4}{2}} = 9^2 = 81$$

Rationalizing the Denominator

📌 **Rule:**

$$\frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

✅ To remove a radical from the denominator, multiply by its conjugate or itself.

📌 **Example:**

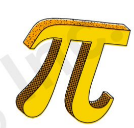
$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Mathematicians prefer not to leave a radical in the denominator

Remember that multiplying both the numerator and the denominator of a fraction by the same number does not change its value.

Area of a Circle

📌 **Formula:**



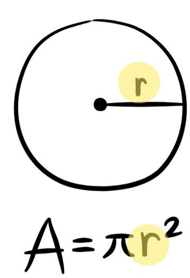
$$A = \pi r^2$$

✅ The area of a circle is found by squaring the radius and multiplying by π.

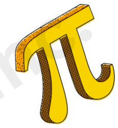
📌 **Example:**

For a circle with radius 3 cm:

$$A = \pi \times 3^2 = 9\pi \approx 28.27 \text{ cm}^2$$



Circumference of a Circle



📌 **Formula:**

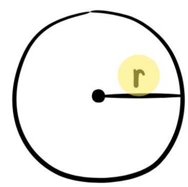
$$C = 2\pi r$$

✅ The circumference of a circle is the total distance around it.

📌 **Example:**

For a circle with radius 4 cm:

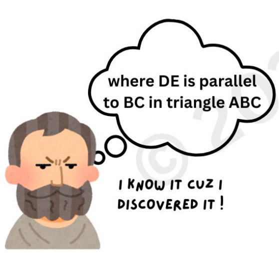
$$C = 2\pi \times 4 = 8\pi \approx 25.13 \text{ cm}$$

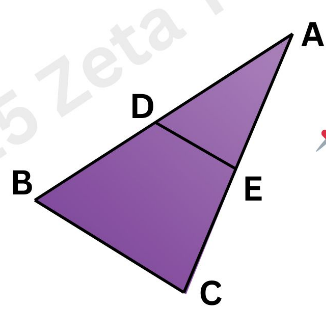


Thales' Theorem

Thales' Theorem (Basic Proportionality Theorem)

If a line is drawn parallel to one side of a triangle and intersects the other two sides, it divides them proportionally.





📌 **Mathematical Formula:**

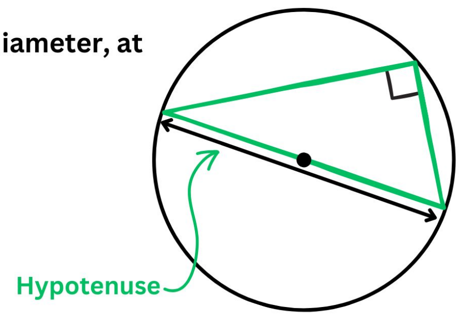
$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

Thales' Theorem (Right Triangle in a Semicircle)

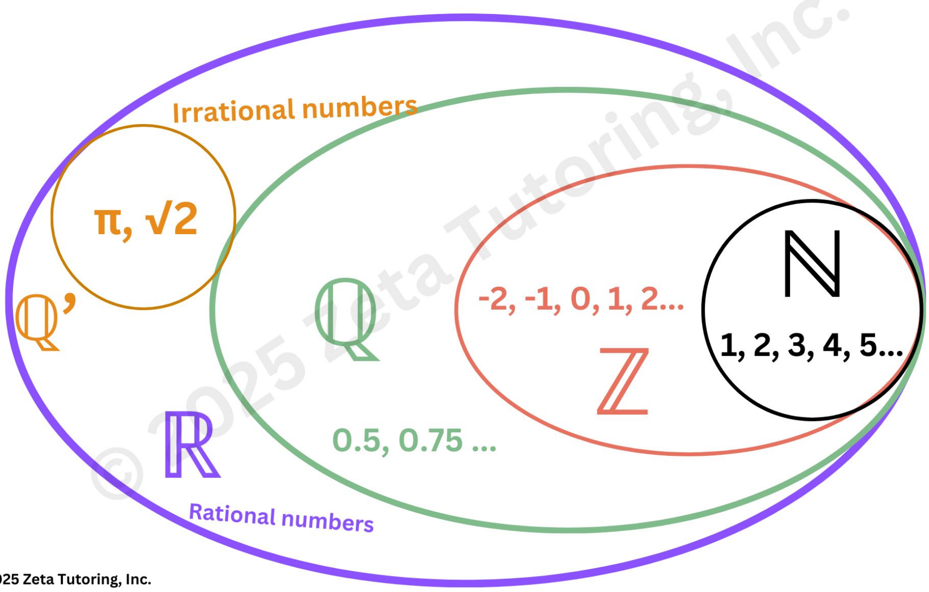
If a triangle is inscribed in a semicircle with one side as the diameter, the triangle is always a right triangle.

- The hypotenuse is the diameter of the semicircle.
- The right angle is always opposite the diameter, at any point on the semicircle.

Hypotenuse = Diameter



Set of Real numbers ℝ



Real Numbers (ℝ) – Explanation

Real numbers (ℝ) include **all numbers** that can be found on the number line. This set consists of **both rational and irrational** numbers.

Natural Numbers (ℕ): Positive counting numbers → {1, 2, 3, 4, ...}

Whole Numbers: Natural numbers including 0 → {0, 1, 2, 3, ...}

Integers (ℤ): Whole numbers including negative values → {... -3, -2, -1, 0, 1, 2, 3, ...}

Rational Numbers (ℚ): Numbers that can be expressed as a fraction $\frac{a}{b}$ where a and b are integers, and b≠0. Example: 0.5

Irrational numbers are numbers that cannot be written as a simple fraction. Their decimal expansions are non-terminating and non-repeating (no specific pattern in which the numbers follow each other). Examples include π and √2. Try it on your calculator !

π = 3.14159

Flashcard for the PEMDAS Rule

PEMDAS Rule (Order of Operations):

It's a rule that determines the correct sequence to solve mathematical expressions.

First, solve any operations inside parentheses. Next, handle any exponents. Then, perform multiplication and division from left to right. Finally, carry out addition and subtraction from left to right.

PEMDAS stands for:

P → Parentheses ( )

E → Exponents X^2

MD → Multiplication & Division (from left to right)

AS → Addition & Subtraction (from left to right)

Flashcard for the PEMDAS Rule

Example Calculation:

Solve: $5 + 3 \times (8 - 6)^2 \div 2$

Step 1: Parentheses → $8 - 6 = 2$

Step 2: Exponents → $2^2 = 4$

Step 3: Multiplication & Division (Left to Right) → $3 \times 4 = 12$, then $12 \div 2 = 6$

Step 4: Addition → $5 + 6 = 11$

Final Answer: 11 ✅

Alternate Interior Angles & Vertically Opposite Angles

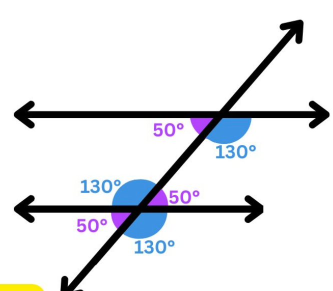
1 Alternate Interior Angles

- Formed when two **parallel lines** are cut by a transversal.
- They are located **on opposite sides** of the transversal and **inside** the parallel lines.
- **Property: They are equal** (if the lines are parallel).

2 Vertically Opposite Angles

Formed when **two lines intersect**.

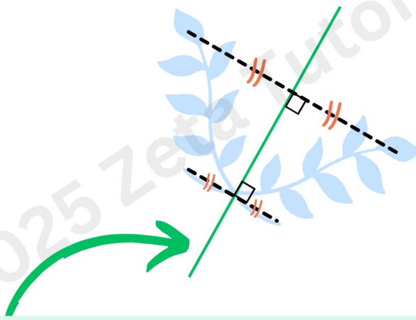
They are opposite to each other across the intersection.



Property: They are always equal.

Axial Symmetry (Reflectional Symmetry)

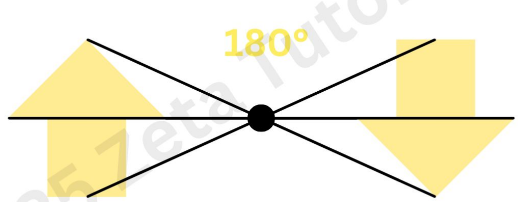
A figure has axial symmetry if one half is the mirror image of the other across a line of symmetry.



The line of symmetry acts as a mirror where every point has a symmetric counterpart on the opposite side.

Central Symmetry (Rotational Symmetry of 180°)

The center of symmetry is the midpoint between every point and its symmetric counterpart.



A figure has central symmetry if rotating it by 180° around a central point keeps it unchanged.

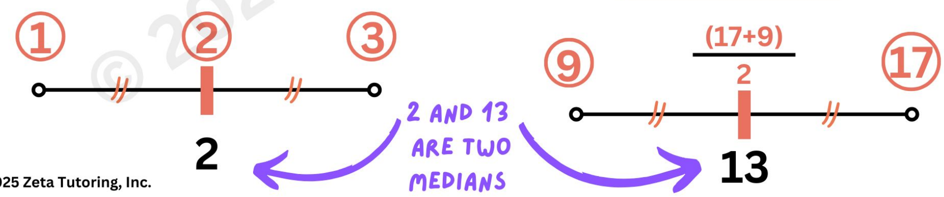
Data: Median & Regression

Median

The median is the middle value of an ordered dataset (sorted from smallest to largest).

If the number of values is odd, the median is the middle number.

If the number of values is even, the median is the average of the two middle numbers.



Data: Median & Regression

Regression (Line of Best Fit)

Regression is a mathematical method used to find the relationship between two variables.

The line of best fit (or regression line) is a line that best approximates the trend in the data.

Price Changes: Discount & Price Increase

Y is the final price of the product

x is the initial price of the product

α is the percentage of increase / decrease (decimal form)

Discount (Price Reduction)

$$Y = (1 - \alpha)x$$

<u>Discount Example:</u>
A jacket costs $80 and has a 25% discount:

$$Y = (1 - 25/100)80$$

$$Y = \$60$$

Price Increase (Markup or Inflation)

$$Y = (1 + \alpha)x$$

<u>Price Increase Example:</u>
A laptop originally costs $500, and the price increases by 12%:

$$Y = (1 + 12/100)500$$

$$Y = \$560$$

Simple & Compound Interest

Simple Interest

Interest earned or paid only on the initial principal (starting amount).

📌**Formula:**

$$A = P(1 + rt)$$

- A = Final amount
- P = Principal (initial amount)
- r = Interest rate (decimal form)
- t = Time (years)

Simple & Compound Interest

Simple Interest

💡 **KEY TAKEAWAY:**
SIMPLE INTEREST IS BETTER FOR SHORT-TERM LOANS (FIXED INTEREST).

📌 **Example:**

A bank offers 5% simple interest on a $1,000 deposit for 3 years:

$$A = 1000(1 + 0.05 \times 3) = 1000(1.15) = 1150$$

Final amount: $1,150

Simple & Compound Interest

Compound Interest:

Interest earned on both the principal and previously earned interest.

📌**Formula:**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- A = Final amount
- P = Principal
- r = Interest rate (decimal)
- n = Number of times interest is applied per year
- t = Time (years)

Simple & Compound Interest

Compound Interest:

💡 **KEY TAKEAWAY:**
COMPOUND INTEREST IS BETTER FOR INVESTMENTS (EXPONENTIAL GROWTH)

📌**Compound Interest Example:**

A savings account earns **5% annual interest**, **compounded monthly** (n = 12), with an initial deposit of $1,000 for 3 years:

$$A = 1000\left(1 + \frac{0.05}{12}\right)^{12\times 3}$$

$$A = 1000 \times (1.00417)^{36} \approx 1161.62$$

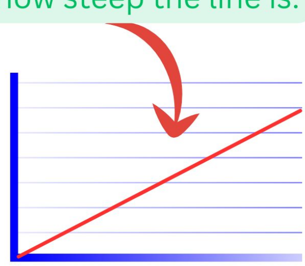
Linear Functions (First-Degree Functions)

A linear function is a function of the form:

$$y = mx + b$$

m = Slope (rate of change)

Slope (m) measures how steep the line is.



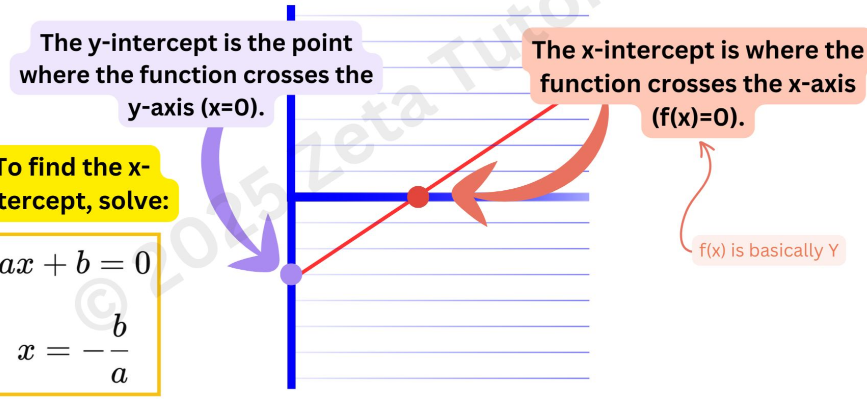
📌**Formula:**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Linear Functions (First-Degree Functions)

Y = mx + b

b = Y-intercept (where the line crosses the y-axis)



To find the x-intercept, solve:

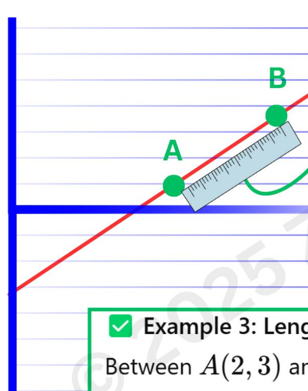
$$ax + b = 0$$

$$x = -\frac{b}{a}$$

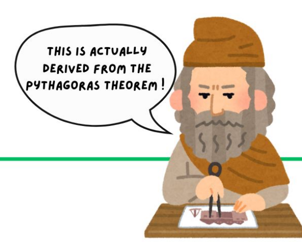
Linear Functions (First-Degree Functions)

Length of a Segment on a Line:

Distance between two points A(X1, Y1) and B(X2, Y2):



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



✅ Example 3: Length of Segment

Between $A(2,3)$ and $B(6,7)$:

$$d = \sqrt{(6-2)^2 + (7-3)^2} = \sqrt{16+16} = \sqrt{32} \approx 5.66$$